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On the One Loop Corrections to Inflation and the CMB Anisotropies

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Abstract

We investigate the one loop effective potential of inflation in a standard model of chaotic inflation. The leading one loop corrections to the effective inflaton potential are evaluated in the quasi de Sitter background, and we estimate the one loop correction to the two-point function of the inflaton perturbations in the Hartree approximation. In this approximation, the one loop corrections depends on the total number of e-foldings of inflation and the maximal effect is estimated to be a correction to the power spectrum of a few percent. However, such a correction may be difficult to disentangle from the background in the simplest scenario.

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1 Introduction

The measurements of the Cosmic Microwave Background (CMB) anisotropies proceed with increasing accuracy, which together with Super Novae (SN) and large scale structure (LSS) data yields an ever more detailed picture of the evolution of the universe. Inflation has earned its place in this picture as a successful paradigm, which, in addition to solving the theoretical problems of the standard Big-Bang theory, explains a number of observational data [1]. However, we know little about the theory of inflation itself, essentially due to our ignorance of physics at the very short distance scales. In fact, even the energy scale of inflation is at present unknown within 10 orders of magnitude. Future experiments might change that situation dramatically. Especially a detection of primordial gravitational waves could pin down the energy scale at which inflation takes place, and as we measure the spectrum of CMB perturbations to higher and higher accuracy we will soon be able to discriminate between different models of inflation and rule out some of the popular ones [2]. This is especially exciting because it is one of our few windows to fundamental physics well above the TeV scale.

Naturally there is a large interest in the theoretical limitations to what we can learn about the physics of inflation and thus physics at very short distance scales from CMB measurements. In addition to attempts to reconstruct parts of the effective potential of inflation from data [3], it has also been attempted to find possible signatures of the UV scale where the effective theory of inflation breaks down, using modified dispersion relations [4], an effective minimum length [5], non-commutativity [6], a new-physics hyper-surface [7], or effective field theory [8–11], combined with constraints from more theoretical considerations [12]. While these considerations are important, it is not yet clear whether such effects are relevant in the real universe.

Here, we investigate the effect of one loop corrections to inflation. If inflation has lasted very long, the Hubble rate at the beginning of inflation H_i , is a new UV scale in between the Planck scale M_p and the Hubble scale, H , when observable modes exits the horizon. The physical modes corresponding to length scales smaller than $1/H_i$, initially smaller than the inflating patch, are pushed outside the horizon during inflation. When integrated over in the loops, they might lead to a significant enhancement of the one loop effects [13–21]. The physical IR cutoff, given by the initial physical radius of the inflating patch, is approximately equivalent to the apparent particle horizon during inflation, and much larger than the Hubble radius. This is the standard choice in the literature [13–21]. We expect that due to causality, to a local observer such IR contributions can never uniquely be identified to be due inhomogeneities, but will look very similar to an addition to the background parameters of the effective theory and thus only indirectly influence our measurements. On the other hand, the effects are sensible to the UV scale of the theory, which is the Hubble rate at the beginning of inflation.

As an example, we will evaluate the one loop corrections to the effective potential and the two-point function in a specific $\lambda\phi^4$ type of chaotic inflation. We estimate that the maximal effect in this particular model is a 1% correction to the power spectrum.

In the first part of the paper we will consider the effective one loop potential and one loop corrections to the background parameters of the theory. The effect appears to be maximally about 1%. In the second part of the paper, we estimate the one loop correction to the two-point function of the quantum fluctuations in the Hartree approximation. We find a similar effect of order 1%. This will lead to a small correction to the slow-roll parameters and a possible change in the cosmological scalar-to-tensor perturbation ratio, which is often also expressed in the so called consistency relation.

Some of the earliest work on the one loop effects in inflation is the work of Vilenkin [13], who studied the effective potential of inflation in the approximation of an exact de Sitter background. Some similar techniques to those applied in the second half of the present paper, was applied in [10] in order to understand the effect of a non-standard vacuum state on the UV cutoff on the one loop effects*. Finally, the effective potential of inflation has been calculated in a slow-roll approximation in [24, 25]. They found, as expected on dimensional grounds, that in an effective field theory of inflation, one loop effects are suppressed by a factor H^2/M_p^2 [8], where H is the Hubble-rate when the observable modes exit and M_p is the reduced Planck scale. Our approach here is somewhat similar in spirit. However, the effects we find are enhanced by a factor of H_i^4/H^4 , where H_i is the Hubble-rate at the beginning of inflation. This is because we go beyond the approximation of a constant H . Our results are consistent with those of [25], when we take $H_i \rightarrow H$.

Thus, it is not always that H^2/M_p^2 is the correct expansion parameter for quantum effects in the effective field theory. In long inflation there could be a significant enhancement, because the loops are effectively suppressed by only a factor of $H_i^4/(H^2 M_p^2)$.

One might note, already in [26–28] it was shown that even within the effective field theory approach, one can have significant enhancements of quantum effects compared to the dimensional H^2/M_p^2 estimate from potential bumps or other non-adiabatic effects during inflation.

In the next subsection we will introduce the uniform curvature gauge, which we will chose for our analysis in the next sections. In section 2, we calculate the one loop corrections to the equation of motion of the inflaton and the quantum corrections to the slow-roll parameters. Our analysis is to first order in the slow-roll and effective field theory expansion. In section 3, we estimate the quantum corrections to the two point function of the inflaton fluctuations in the Hartree approximation. In section 4, we discuss our findings.

1.1 Uniform curvature gauge

We find that it is simplest to understand the one loop correction to the inflaton potential in the uniform curvature gauge, because here the residual degrees of the freedom of the perturbations can be identified with the inflaton field fluctuations. This makes it more transparent to understand the perturbations as quantum fluctuations of the inflaton and to renormalize the divergences with simple counter terms in the renormalized inflaton potential.

*An other approach to loop effects, the stochastic approach, has been applied by Linde [22] (see also [23]), in order to understand the global structure of space-time in eternal inflation.

In order to calculate the renormalized two point function $\langle \delta\phi^2 \rangle_0$ in the effective theory, we will thus write the perturbed metric to first order in the uniform curvature gauge where the metric takes the form [29, 30]

$$ds^2 = -(1 + 2\varphi)dt^2 + 2aB_{,i}dtdx^i + a^2\delta_{ij}dx^i dx^j . \quad (1)$$

In the absence of anisotropic stress $\dot{B} + 2HB + 2\varphi/a = 0$, we expect that there should be only one independent scalar degree of freedom like in the longitudinal gauge if we have fixed the gauge correctly. In fact one finds using the constraints from the Einstein equations that the equation of motion for the inflaton perturbations becomes

$$\ddot{\delta\phi} + 3H\dot{\delta\phi} - \frac{1}{a^2}\nabla^2\delta\phi + (V_{\phi\phi} - 6\epsilon H^2)\delta\phi = 0 , \quad (2)$$

to first order in the slow-roll parameters. This shows that in this gauge the quantum fluctuations can be identified with the gauge invariant Sasaki-Mukhanov variable, Q , which satisfies the same equation [31]. In appendix A, we have generalized this to third order in perturbations and computed the effective action of the perturbations to fourth order, since we will need it to compute the self-interactions of the quantum fluctuations of the inflaton.

Thus, in the zero curvature gauge we can describe the generation of perturbations from quantum fluctuations of the inflaton in Fourier space, by expanding the quantum field $\delta\phi(t, \mathbf{x})$ in c-number mode functions with respect to the Bunch-Davis vacuum

$$\delta\phi(t, \mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left[U_k(t)e^{i\mathbf{k}\cdot\mathbf{x}}a_{\mathbf{k}} + U_k^*(t)e^{-i\mathbf{k}\cdot\mathbf{x}}a_{\mathbf{k}}^\dagger \right] , \quad (3)$$

where the operator $a_{\mathbf{k}}$ annihilates the Bunch-Davis vacuum $|0\rangle_0$. The mode functions satisfy the Fourier transform of eq.(2), which in conformal coordinates yields

$$\left[\eta^2 \frac{\partial^2}{\partial \eta^2} - 2\eta \frac{\partial}{\partial \eta} + \eta^2 k^2 + \frac{V_{\phi\phi} - 6\epsilon H^2}{H^2} \right] U_k(\eta) = 0 , \quad (4)$$

and the conformal time is defined as $a(\eta)d\eta = dt$. So, with the usual normalization to the Minkowski vacuum in the infinite past the solution for the modes becomes

$$U_k(\eta) = \frac{\sqrt{\pi}}{2} H \eta^{3/2} H_\nu^{(2)}(k\eta) , \quad (5)$$

where $H_\nu^{(2)}(k\eta)$ is the usual second Hankel function and $\nu = 3/2 + 3\epsilon - \eta$. The spectral index of the perturbations is defined as

$$\mathcal{P}_{\delta\phi}(k, t) = \frac{k^3}{2\pi^2} \langle |\delta\phi_k(t)|^2 \rangle_0 = \frac{k^3}{2\pi^2} |U_k(t)|^2 , \quad (6)$$

which on super-Hubble scales approximately gives

$$\mathcal{P}_{\delta\phi}(k, t) \simeq \frac{H^2}{4\pi^2} \left(\frac{k}{aH} \right)^{n-1} , \quad (7)$$

with $n - 1 = 3 - 2\nu$. It is important to note that this solution is to first order in the slow-roll parameters and treating them as constant.

2 One loop effective field theory of inflation

Inflation is most often formulated in terms of a minimally coupled scalar inflaton quantum field, ϕ , with a Lagrangian

$$\mathcal{L} = -\frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) , \quad (8)$$

in a quasi de Sitter space with metric

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j , \quad (9)$$

where the scale factor is conveniently written on the form $a(t) = \exp(Ht)$ and the Hubble rate H is almost constant, a statement which we can quantify in terms of the slow-roll parameter $\epsilon = -\dot{H}/H^2 \ll 1$.

For definiteness we will consider a simple version of chaotic inflation driven by an inflaton field with potential

$$V(\phi) = \frac{1}{4}\lambda\phi^4 . \quad (10)$$

The resulting equation of motion is

$$\square_g \phi + \lambda\phi^3 = 0 . \quad (11)$$

The dynamics is conveniently analyzed in an effective field theory for the classical background field ϕ_c , defined by

$$\phi = \phi_c + \delta\phi , \quad (12)$$

where $\delta\phi$ is a quantum field with a vanishing vacuum expectation value enforced through the tadpole condition

$$\langle \delta\phi \rangle_0 \equiv \langle 0 | \delta\phi | 0 \rangle = 0 , \quad (13)$$

such that

$$\langle \phi \rangle_0 \equiv \langle 0 | \phi | 0 \rangle = \phi_c . \quad (14)$$

From Wick's theorem it follows that also $\langle \delta\phi^3 \rangle_0 = 0$. The effective equation of motion for the classical field ϕ_c can then be obtained by taking the expectation value of the equation of motion in eq. (11), which yields

$$\langle \square_{g+\delta g} \phi \rangle_0 + 3\lambda \langle \delta\phi^2 \rangle_0 \phi_c + \lambda\phi_c^3 = 0 . \quad (15)$$

To calculate $\langle \square_g \phi \rangle_0$, we need to take into account also the metric perturbations in the uniform curvature gauge as discussed in the introduction. The constraint equations relating the scalar metric perturbations to the scalar field fluctuations are in general complicated. Thus, we find that the effective equation of motion to second order, is most easily computed from the effective action of the perturbations expanded to third order using the ADM formalism [34]. In the appendix we have in fact derived the effective action of the perturbations to fourth order, which we will need later. The expressions are given in eq.(74) and eq.(75).

Using the linear perturbation equation eq.(2), we find from eq.(74), on super-Hubble scales where we can neglect gradient terms, to leading order

$$\ddot{\phi}_c + 3H\dot{\phi}_c + 6\lambda \langle \delta\phi^2 \rangle_0 \phi_c + \lambda\phi_c^3 = 0 . \quad (16)$$

The contributions from sub-Hubble scales, contributing to the trace-anomaly, were computed in [25]. However, since they are not amplified by long inflation, those contributions are always suppressed by a factor H^2/M_p^2 and we will therefore ignore gradient terms and second order time derivatives of the perturbations. Thus, to obtain an order of magnitude estimate of the minimal one loop effects, it should be sufficient to consider only the contributions in eq.(16). In section 3.2 we will see more formally how the effective one loop equation of motion in eq.(16) follows from the tadpole renormalization condition of the quantum fluctuations.

From the effective equation of motion in eq.(16), we can see that the effective mass gets a one loop contribution

$$\delta m_{eff}^2 = 6\lambda \langle \delta\phi^2 \rangle_0 . \quad (17)$$

The effective potential, which is consistent with the effective energy density derived from the effective energy-momentum tensor computed in [15, 16], is

$$V_{eff}(\phi_c) = V(\phi_c) + 2V' \langle \delta\phi\varphi \rangle_0 + \frac{1}{2}V'' \langle \delta\phi^2 \rangle_0 . \quad (18)$$

The inflationary expansion is then given by the slow-roll of the classical field in the effective potential, and in the slow-roll approximation the effective Hubble rate is determined by the effective Friedman equations

$$3H^2 \simeq \frac{1}{M_p^2} V_{eff}(\phi_c) , \quad 3H\dot{\phi}_c \simeq -V'(\phi_c) - \delta m_{eff}^2 \phi_c , \quad (19)$$

as can be seen from the effective equation of motion eq.(16), with $\ddot{\phi}_c \simeq 0$. The effective slow-roll parameters receive a one loop correction $\delta\epsilon_{eff}$, $\delta\eta_{eff}$, when compared to the tree-level slow-roll parameters ϵ , η , such that

$$\epsilon_{eff} = \epsilon + \delta\epsilon_{eff} , \quad \eta_{eff} = \eta + \delta\eta_{eff} . \quad (20)$$

With the following definition of the slow-roll parameter,

$$\epsilon_{eff} \equiv \frac{1}{2M_p^2} \frac{\dot{\phi}_c^2}{H^2} , \quad (21)$$

we can compute

$$\delta\epsilon_{eff} \simeq \frac{4}{3} \frac{\delta m_{eff}^2}{H^2} , \quad (22)$$

and we assumed that the one loop contribution to the effective potential is small compared to the tree-level value, i.e. $\delta m_{eff}^2 \phi_c^2 \ll V(\phi_c)$.

To make the approximations self-consistent, we should use the effective slow-roll parameters in eq.(20) to calculate the quantum fluctuations in eq.(2). This is similar to the *cactus* or *Hartree* approximation [13], which corresponds to an infinite summation of self-energy diagrams of the form shown in fig.(1). In section 3, we shall calculate explicitly the one loop diagram in fig.(1), which is first order in λ and similar to computing the loop correction to η using Dyson's equation. To second order in λ there are also other one loop diagrams, not included in the cactus approximation, that contributes, since the vacuum expectation value of the background field is non-vanishing.

Assuming, according to the standard lore, that the initial vacuum is the Euclidean vacuum, the spectral index n_{eff} of the primordial scalar curvature perturbations generated during inflation follows

$$n_{eff} - 1 = 2\eta_{eff} - 6\epsilon_{eff} , \quad (23)$$

as we will discuss in more details in the next section where we will also calculate the magnitude of the corrections. Especially we need to calculate $\langle \delta\phi^2 \rangle_0$ and extract the dominant finite part.

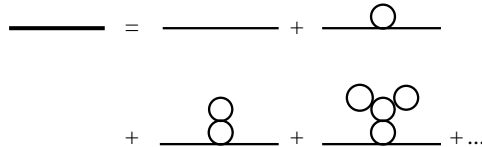


Figure 1: The self-energy diagrams in the cactus approximation.

2.1 Renormalized quantum fluctuations

If one assumes that the inflationary state is preceded by a radiation dominated one, then the expectation value $\langle \delta\phi^2 \rangle_0$ will receive two contributions. A thermal contribution and a vacuum contribution. As inflation kicks in, only the vacuum contribution will grow, and one can ignore the thermal contribution. It is the modes corresponding to physical wavelengths which are stretched to super-Hubble scales by the inflationary expansion, that are responsible for the growth. As modes are continuously pushed outside the horizon, they freeze and give a little accumulating contribution to the expectation value. Thus, the modes that were already outside the horizon at the beginning of inflation, gives only a constant contribution, which one can ignore a while after inflation has started.

Above, we calculated the one loop induced mass in terms of $\langle \delta\phi^2 \rangle_0$, which we can now compute directly from the power spectrum

$$\langle |\delta\phi(t, \mathbf{x})|^2 \rangle_0 = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \langle |\delta\phi_k(t)|^2 \rangle_0 = \int_{a_i H_i}^{a\Lambda} \frac{dk}{k} \mathcal{P}_{\delta\phi}(k, t) . \quad (24)$$

The *IR* cutoff on comoving momenta $a_i H_i$ is dynamically given by the scale that exits the horizon at the beginning of inflation, as explained above, and the *UV* cutoff has been

introduced by hand to regulate a quadratic UV divergence, which should be subtracted by a proper renormalization. When the UV divergences has been subtracted, one finds that $\langle \delta\phi^2 \rangle_0$ is dominated by the IR modes.

In order to actually evaluate the integral, we will split it in an IR part and a UV part that we will evaluate separately

$$\langle |\delta\phi(t, \mathbf{x})|^2 \rangle_0 = \langle |\delta\phi(t, \mathbf{x})|^2 \rangle_{IR} + \langle |\delta\phi(t, \mathbf{x})|^2 \rangle_{UV} , \quad (25)$$

with

$$\langle |\delta\phi(t, \mathbf{x})|^2 \rangle_{IR} = \int_{a_i H_i}^{a_0 H_0} \frac{dk}{k} \mathcal{P}_{\delta\phi}(k, t) , \quad \langle |\delta\phi(t, \mathbf{x})|^2 \rangle_{UV} = \int_{a_0 H_0}^{a\Lambda} \frac{dk}{k} \mathcal{P}_{\delta\phi}(k, t) . \quad (26)$$

Here we let a_0, H_0 denote the values of a, H at the time when the relevant scales for the CMB crosses outside the horizon.

Let us first evaluate the more important IR part. To properly account for the running of the spectral index with the scale, we will solve the mode function, U_k , in pure de Sitter, but using the value of $H = H_k$ and $\dot{\phi} = \dot{\phi}_k$ when the given mode crosses the horizon. If we specify to the specific $\lambda\phi^4$ model of chaotic inflation, this implies that we obtain the right k scaling from (See appendix B)

$$U_k(\eta) = \left(\frac{H_k}{H(\eta)} \right)^{3/2} U_k^{ds}(\eta) = \frac{\sqrt{\pi}}{2} H \eta^{3/2} \left(\frac{H_k}{H(\eta)} \right)^{3/2} H_{3/2}^{(2)}(k\eta) , \quad (27)$$

where the mode solution in pure de Sitter was denoted by $U_k^{ds}(\eta)$.

If we let $N = \ln(a/a_i)$ denote the number of e-foldings of expansions since the beginning of inflation, it is convenient to recast the relevant integral into an integral over N . To do so, we use the relations $d \ln k = dN$, $\ln(aH/k) = N$ and

$$H_k^2 = \frac{\lambda}{12M_p^2} (\phi_i^2 - 64M_p^2 N)^2 . \quad (28)$$

Now we can easily evaluate the dominant part of the IR contribution to the relevant correlator, we obtain from eq.(83) in appendix B

$$\langle |\delta\phi(t, \mathbf{x})|^2 \rangle_{IR} \simeq \frac{H^2}{16\pi^2} \left(\lambda \frac{1024 M_p^2}{3 H^2} \right)^{3/2} N^4 . \quad (29)$$

If we had used same type of argument for a $m^2\phi^2$ theory, the IR part of the power spectrum would instead of $H_k^{3/2}$ scale as H_k^2 . If one applies the relation $\varphi = -\sqrt{\epsilon/2}\delta\phi/M_p$, the result is consistent with [15–17].

The UV part is much easier to evaluate. Inside the horizon, the modes are not sensitive to the details of the expansion

$$\begin{aligned} \langle |\delta\phi(t, \mathbf{x})|^2 \rangle_{UV} &\simeq \frac{1}{8\pi} H^2 \int_1^{\Lambda_H} \frac{dp_H}{p_H} p_H^3 \left| H_{3/2}^{(2)}(p_H) \right|^2 \\ &= \frac{1}{8\pi^2} H^2 (\Lambda_H^2 + \ln(\Lambda_H^2)) . \end{aligned} \quad (30)$$

where $p_H = |k/(aH)| = |k\eta|$.

Finally we can add the *IR* and the *UV* contributions to obtain the full correlator to leading order

$$\langle |\delta\phi(t, \mathbf{x})|^2 \rangle_0 \simeq \frac{1}{8\pi^2} H^2 (\Lambda_H^2 + \ln(\Lambda_H^2) + \Delta_N) , \quad (31)$$

where we for convenience have defined

$$\Delta_N = \frac{1}{2} \left(\lambda \frac{1024}{3} \frac{M_p^2}{H^2} \right)^{3/2} N^4 . \quad (32)$$

Another way of writing Δ_N , which is illuminating, is in terms of the Hubble rate at the beginning of inflation H_i , which gives

$$\Delta_N = \frac{\sqrt{3}}{64} \frac{1}{\sqrt{\lambda}} \frac{H_i^4}{M_p H^3} . \quad (33)$$

The *UV* divergent parts are canceled in the renormalized potential introducing appropriate counter-terms. We shall not elaborate on this aspect, but note that the physical *IR* contributions survives as the dominant one loop correction to the effective renormalized potential after subtracting the *UV* divergences, as discussed in more details in [24].

2.2 Quantum corrections to slow-roll parameters

We can now evaluate the quantum corrections to the slow-roll parameters and the spectral index of the CMB anisotropies, using the relations derived in eq.(17) and eq.(22). The corrections to the slow-roll parameters can then be written in terms of Δ_N in the following way

$$\delta\epsilon_{eff} = \frac{\lambda}{\pi^2} \Delta_N . \quad (34)$$

From eq.(33) we obtain

$$\delta\epsilon_{eff} = \frac{\sqrt{3}}{64\pi^2} \sqrt{\lambda} \frac{H_i^4}{M_p H^3} . \quad (35)$$

If we use the background values for the slow-roll parameters

$$\epsilon = \sqrt{\frac{16\lambda}{3}} \frac{M_p}{H} , \quad (36)$$

we can easily write the effective slow-roll parameter ϵ_{eff} in terms of a fractional correction to the background value

$$\epsilon_{eff} = \epsilon \left(1 + \frac{\delta\epsilon_{eff}}{\epsilon} \right) = \epsilon \left(1 + \frac{3}{256\pi^2} \frac{H^2}{M_p^2} \frac{H_i^4}{H^4} \right) . \quad (37)$$

From an effective field theory point of view we generically expect that corrections are of order H^2/M_p^2 [8, 24]. In fact, if we take $H_i \rightarrow H$ that is exactly the type of correction we find. However, due to the variation in the expansion rate over time, the effect is amplified by the potentially large term H_i^4/H^4 . It is also interesting to note that

$$\frac{H_i^4}{M_p^4} \simeq 10^5 \lambda^2 N^4, \quad (38)$$

so the correction depends very sensitively on the total number of e-foldings. This leads us to the possibility that the spectral index carries a tiny imprint from the beginning of inflation through the loop effects, such that we in principle indirectly can observe the total number of e-foldings of inflation in this model.

In the model of chaotic inflation with a potential $V(\phi) = 1/4\lambda\phi^4$, in the regime when $V(\phi) > M_p^4$ quantum gravity effects are large and no classical description of space is possible within the effective field theory framework. When $10^{-4}M_p^4 \lesssim V(\phi) \lesssim M_p^4$, the amplitude of inflaton fluctuations is large compared to the mean and the perturbations can not be treated perturbatively. This is the self-reproduction regime. For our effective field theory description of inflation to be consistent, we must therefore at least require that the energy density of inflation, ρ , is safely below the Planck scale,

$$\rho^{1/4} \lesssim 0.1 M_p. \quad (39)$$

This implies that the Hubble rate at the beginning of inflation must satisfy $H_i^4 \lesssim 10^{-9} M_p^4$. If we assume that the Hubble rate, when the observable modes left the horizon, is given by $H \simeq 10^{-5} M_p$, then the maximal correction to the first slow-roll parameter, ϵ , is

$$\frac{|\delta\epsilon_{eff}|}{\epsilon} \lesssim \frac{30}{256\pi^2} \simeq 0.01. \quad (40)$$

Thus, the maximal correction to the slow-roll parameter, ϵ , is less than about 1% in this approximation.

3 One loop contributions to the power spectrum

In order to evaluate the one loop contributions self-consistently, we need to compute the one loop contribution to the two-point function of the quantum fluctuations. It is convenient to apply the Schwinger-Keldysh formalism [32, 33] for this purpose. Below, we will first briefly review the Schwinger-Keldysh formalism and then calculate the two-point function in the Hartree approximation. This is similar to computing the one loop correction to the second slow-roll η_{eff} using Dyson's equation.

3.1 Schwinger-Keldysh formalism

This formalism has recently been reviewed and applied to the specific problem of a self-interacting scalar field in de Sitter space, in order to understand the UV properties of

inflaton fluctuations originating in an alpha vacuum state [10]. Here we will therefore only review it briefly in order to introduce the proper notation, which will essentially follow [10].

The Schwinger-Keldysh formalism is a perturbative approach for solving the evolution of a matrix element over a finite time interval. This is useful in the inflationary coordinates of de Sitter space, where there is no well defined asymptotic *in* and *out* state in which to define an *S*-matrix, essentially due to the explicit time-dependence of the metric. Instead of specifying an initial state in the infinite past, one develops a given state forward in time from a specified initial time η_{infl} , which we can think of as being the beginning of inflation.

In the interaction picture, the evolution of operators is given by the free Hamiltonian, H_0 , while the part of the Hamiltonian containing the interactions, H_I , is used to evolve the states in the theory. If one specifies a density of state $\rho(\eta_{infl})$ at a specific moment in time, then one can define a unitary time evolution operator $U_I(\eta, \eta')$ that evolves the state, and which is given by Dyson's equation

$$U_I(\eta, \eta_{infl}) = T \left\{ e^{-i \int_{\eta_{infl}}^{\eta} d\eta' H_I(\eta')} \right\} , \quad (41)$$

where T is the time ordering of the product in the curly brackets. Then one can write

$$\rho(\eta) = U_I(\eta, \eta_{infl}) \rho(\eta_{infl}) U_I^{-1}(\eta, \eta_{infl}) . \quad (42)$$

Absorbing a step function $\Theta(\eta - \eta_{infl})$ into the interaction Hamiltonian, H_I , such that interaction only turn on after η_{infl} , one can then write the evolution of the expectation value of some operator, \mathcal{O} , as

$$\langle \mathcal{O} \rangle (\eta) = \frac{\text{Tr} [U_I(-\infty, 0) U_I(0, \eta) \mathcal{O} U_I(\eta, -\infty) \rho(\eta_{infl})]}{\text{Tr} [U_I(-\infty, 0) U_I(\eta, -\infty) \rho(\eta_{infl})]} . \quad (43)$$

This matrix element describes a system in the initial state $\rho(\eta_{infl})$, evolved from conformal time $-\infty$ to 0 with an operator inserted at η , and back again from 0 to $-\infty$. To evaluate this matrix element, one can formally double the field content of the theory, with a set of "+" fields on the increasing-time contour and a set of "-" fields on the decreasing-time contour and then group the evolution operators into a single time-ordered exponential. One can then write the interacting part of the action appearing in Dyson's equation together in a single time-contour, as

$$S_I = - \int_{-\infty}^0 d\eta [H_I(\psi^+) - H_I(\psi^-)] , \quad (44)$$

where contractions between different pairs of the two types of fields now yields four kinds of propagators

$$\begin{aligned} \langle 0 | T [\psi^\pm(x) \psi^\pm(x')] | 0 \rangle &= -i G^{\pm\pm}(x, x') \\ &= -i \int \frac{d^3 k}{(2\pi)^3} e^{i\vec{k} \cdot (\vec{x} - \vec{x}')} G_k^{\pm\pm}(\eta, \eta') . \end{aligned} \quad (45)$$

The time-ordering of the contractions then yields

$$\begin{aligned}
G_k^{++}(\eta, \eta') &= G_k^>(\eta, \eta')\Theta(\eta - \eta') + G_k^<(\eta, \eta')\Theta(\eta' - \eta) \\
G_k^{--}(\eta, \eta') &= G_k^>(\eta, \eta')\Theta(\eta' - \eta) + G_k^<(\eta, \eta')\Theta(\eta - \eta') \\
G_k^{-+}(\eta, \eta') &= G_k^>(\eta, \eta') \\
G_k^{+-}(\eta, \eta') &= G_k^<(\eta, \eta') ,
\end{aligned} \tag{46}$$

where

$$\begin{aligned}
G_k^>(\eta, \eta') &= iU_k(\eta)U_k^*(\eta') \\
G_k^<(\eta, \eta') &= iU_k^*(\eta)U_k(\eta') .
\end{aligned} \tag{47}$$

One can of course also define $G^>(x, x')$, $G^<(x, x')$ from which $G_k^>(\eta, \eta')$, $G_k^<(\eta, \eta')$ can be obtained by a Fourier transform.

From Dyson's equation eq.(41) and eq.(44), one finds that eq.(43) yields [10]

$$\langle 0 | \mathcal{O} | 0 \rangle = \frac{\langle 0 | T \left\{ \mathcal{O} e^{-i \int_{-\infty}^0 d\eta [H_I(\phi_c, \psi^+) - H_I(\phi_c, \psi^-)]} \right\} | 0 \rangle}{\langle 0 | T \left\{ e^{-i \int_{-\infty}^0 d\eta [H_I(\phi_c, \psi^+) - H_I(\phi_c, \psi^-)]} \right\} | 0 \rangle} , \tag{48}$$

if the initial state is the vacuum state $|0\rangle$. Since we absorbed the step function $\Theta(\eta - \eta_{infl})$ in H_I , the time integral effectively have η_{infl} as lower limit.

3.2 Hartree approximation

In appendix A, we have expanded the action for the inflaton field fluctuations to fourth order. From the action given in eq.(74) and eq.(75), it is simple to compute the interaction Hamiltonian. Like in section 2, we shall constrain ourselves for simplicity, to consider only interactions that do not contain space derivatives. These are the interactions that we naively expect can give a large one loop contribution after a long period of inflation. After using the results in eq.(74), eq.(75), some partial integrations, and applying the linear perturbation equation in eq.(2), we obtain with $\psi \equiv \delta\phi$, that the effective interaction Hamiltonian in the present approximation can be given approximately as

$$\begin{aligned}
H_I(\phi_c, \psi^\pm) &\simeq \int \frac{d^3y}{\eta^4 H^4} \left[\psi^\pm (\phi_c'' + 2\mathcal{H}\phi_c' + \lambda\phi_c^3) \right. \\
&\quad \left. + 2\lambda\phi_c\psi^{\pm 3}(y) + \frac{15}{4}\lambda\psi^{\pm 4}(y) \right] ,
\end{aligned} \tag{49}$$

in order to estimate the one loop correction to the two-point function of the inflaton field fluctuations. Our equations are consistent with section two. In fact, to first order in λ one can verify that the tadpole renormalization condition [10],

$$\begin{aligned}
0 &= \langle \psi^\pm(x) \rangle_0 \\
&= - \int_{-\infty}^{\eta_0} d\eta \int \frac{d^3\vec{y}}{\eta^4 H^4(\eta)} [(G^>(x, y) - G^<(x, y)) (\phi_c'' + 2\mathcal{H}\phi_c' + \lambda\phi_c^3 - i6\lambda\phi_c G^>(y, y))] \tag{50}
\end{aligned}$$

yields the effective one loop equation of motion in eq.(16).

The two-point function to one loop order can be organized in terms of contributions to zero $T^{(0)}$, first $T^{(1)}$, second $T^{(2)}$ and second order in λ ,

$$\langle \psi^+(\eta_0, \vec{x}_1), \psi^+(\eta_0, \vec{x}_2) \rangle = T^{(0)}(\eta_0, |\vec{x}_1 - \vec{x}_2|) + T^{(1)}(\eta_0, |\vec{x}_1 - \vec{x}_2|) + T^{(2)}(\eta_0, |\vec{x}_1 - \vec{x}_2|) , \quad (51)$$

where $T^{(0)}$ is the lowest order free tree-level contribution to the two-point function. The first order contribution in λ receives a contribution

$$\begin{aligned} T^{(1)}(x_1, x_2) &= i \int_{-\infty}^{\eta_0} d\eta \int \frac{d^3 \vec{y}}{\eta^4 H^4(\eta)} (G^>(x_1, y) G^>(x_2, y) - G^<(x_1, y) G^<(x_2, y)) \\ &\quad \times 45 \lambda G^>(y, y) . \end{aligned} \quad (52)$$

To second order in λ there is two one loop contributions

$$T^{(2)}(x_1, x_2) = T_1^{(2)}(x_1, x_2) + T_2^{(2)}(x_1, x_2) , \quad (53)$$

and three two-loop contributions $\tilde{T}_i(x_1, x_2)$. These diagrams are given in appendix D. Here we will focus on the contributions to linear order in λ .

It is convenient to define the Fourier transform of the diagrams, which is relevant when computing the corrections to the power spectrum

$$T^{(i)}(x_1, x_2) = \int \frac{d^3 k}{(2\pi)^3} e^{i\vec{k} \cdot (\vec{x}_1 - \vec{x}_2)} T^{(i)}(\eta_0, k) . \quad (54)$$

In this notation the one loop corrected power spectrum becomes

$$\mathcal{P}(\eta_0, k) = \frac{k^3}{2\pi^2} \sum_i T^{(i)}(\eta_0, k) . \quad (55)$$

Let us evaluate specifically the diagram which is first order in λ . That is the *seagull* diagram $T^{(1)}(x_1, x_2)$ shown in fig.(2). In Fourier space we obtain

$$T^{(1)}(\eta_0, k) = -2 \int_{\eta_{infl}}^{\eta_0} \frac{d\eta}{\eta^4 H^4} \text{Im} [G_k^>(\eta_0, \eta) G_k^>(\eta_0, \eta)] \times 45 \lambda \int \frac{d^3 k'}{(2\pi)^3} G_{k'}^>(\eta, \eta) \quad (56)$$

To compute the seagull diagram $T^{(1)}(\eta_0, k)$ given by

$$T_2^{(1)}(\eta_0, k) = -90 \lambda \int_{\eta_{infl}}^{\eta_0} \frac{d\eta}{\eta^4 H^4} \text{Im} [U_k^2(\eta) U_k^{*2}(\eta')] \int \frac{d^3 k'}{(2\pi)^3} G_{k'}^>(\eta, \eta) , \quad (57)$$

we have to understand the behavior of the integral. We will first make some approximations, such that we can solve it analytically. Since the physically relevant (observable) k -modes only spent less than 60 e-folds outside the horizon, as a first approximation, we will take

$$U_k(\eta) = \frac{iH}{k\sqrt{2k}} (1 + ik\eta) e^{-ik\eta} . \quad (58)$$

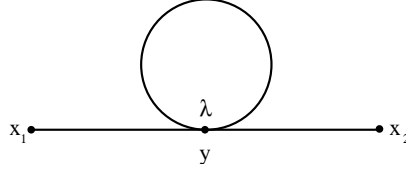


Figure 2: The seagull diagram contributing to first order in λ .

In appendix C we have shown the computation, when the mode function is taken to be given by a Hankel function with index $\nu \neq 3/2$ and reproduced the correct scaling for the seagull contribution. We have shown that we are making a very small error above, when estimating the magnitude of the seagull contribution, by approximating the mode function with the scale invariant one.

Changing variables to $x = -k\eta$ and letting $\eta_{infl} \rightarrow -\infty$, we obtain the following integral

$$T_1^{(1)}(\eta_0, k) = -45 \frac{\lambda}{k^3} \frac{H^2}{4\pi^2} \Delta_N \int_{x_0}^{\infty} \frac{dx}{x^4} [(-1 + x^2 + x_0^2 - x_0^2 x^2 + 4x_0 x) \sin(2(x - x_0)) + 2(-x_0 + x_0 x^2 + x - x_0^2 x) \cos(2(x - x_0))] . \quad (59)$$

Since the integrand falls off as a power on sub-Hubble scales $x \gg 1$, the dominant contribution to the integral is from $x \lesssim 1$ and since

$$\int \frac{d^3 k'}{(2\pi)^3} G_{k'}^>(\eta, \eta) = \frac{H^2}{2\pi^2} \Delta_N , \quad (60)$$

is almost constant on super-Hubble scales for physically observable k -modes, we have approximated it with a constant in the integral in eq.(59). Now the integral is easy to solve and it yields

$$T_1^{(1)}(\eta_0, k) = -45 \frac{\lambda}{k^3} \frac{H^2}{4\pi^2} \Delta_N \left\{ \frac{20}{3} + \left[\frac{1}{3} x_0^2 \pi - \frac{1}{3} \pi + \frac{2}{3} \text{Si}(2x_0) - \frac{2}{3} x_0^2 \text{Si}(2x_0) - \frac{44}{3} x_0 \text{Ci}(2x_0) \right] \sin(2x_0) + \left[-\frac{22}{3} x_0 \pi + \frac{44}{3} x_0 \text{Si}(2x_0) - \frac{2}{3} x_0^2 \text{Ci}(2x_0) + \frac{2}{3} \text{Ci}(2x_0) \right] \cos(2x_0) \right\} . \quad (61)$$

Taking the limit $x_0 \rightarrow 0$, one finds

$$T^{(1)}(\eta_0, k) \simeq -45 \frac{\lambda}{k^3} \frac{H^2}{4\pi^2} \Delta_N \left[\frac{20}{3} + \frac{2}{3} \text{Ci}(2x_0) \right] , \quad (62)$$

which shows that this one loop contribution to first order in λ is constant on super-Hubble scales up to a logarithmic time-dependence, as we would have expected within this approximation.

From eq.(55) we find on super-Hubble scales

$$\mathcal{P}(\eta_0, k) \simeq \frac{H^2}{4\pi^2} \left[1 - \frac{15}{\pi^2} \lambda \Delta_N (10 + \text{Ci}(-2k\eta_0)) + \mathcal{O}(\lambda^2) \right]. \quad (63)$$

To estimate the logarithmic correction to the power spectrum, we can again use $H \simeq 10^{-5} M_p$, $\lambda \simeq 10^{-14}$, $H_i^4 \lesssim 10^{-9} M_p^4$, which implies

$$\frac{45}{3\pi^2} \lambda \Delta_N = \frac{15\sqrt{3}}{16\pi^2} \sqrt{\lambda} \frac{H_i^4}{M_p H^3} \lesssim 0.016. \quad (64)$$

Again we find that within the present approximation, the one loop correction to the power spectrum is maximally of order 1%, if inflation is sufficiently long. We might also note again, that in the limit $H_i \rightarrow H$ the contribution becomes negligible as expected. We conclude that our results are consistent with a maximal effect of a few percent.

4 Discussion

We have calculated the one loop corrections to a $\lambda\phi^4$ type of chaotic inflation. We found that in the effective field theory the quantum corrections are not always suppressed by a factor H^2/M_p^2 as generally expected. In scenarios with very long inflation there can be a significant enhancement of the one loop effects. This is because the phase-space volume of IR modes, to be integrated over in the loops, grows with the total number of e-folds of inflation. Hence, the loops are effectively suppressed only by a factor of $H_i^4/(H^2 M_p^2)$, where H_i is the Hubble rate at the beginning of inflation. We estimated that if inflation is very long, or equivalently the Hubble scale at the beginning of inflation was very close to the Planck scale, the effects can be as large as perhaps a few percent.

Since the effect comes from an integration of IR modes in the loop integrals, we expect that it looks similar to a redefinition of the background parameters of the theory up to a small time dependence. However, there is no reason a priori why a mass term of the same magnitude as the one loop induced mass term should present at tree-level. Such a mass term could affect the slow-roll parameters and the cosmological scalar-to-tensor consistency relation, and one can speculate about the theoretical possibility of an observational indirect evidence for very long inflation.

One aspect which we find also deserves further exploration is if there could be other new effects from an IR enhancement of one loop corrections. For instance if it could lead to an enhancement of the effects found in [4–7, 9, 10]. It is as well tempting to speculate about non-gaussianities, which will also be enhanced by one loop effects in models with long inflation. In an effective $\lambda\phi^4$ theory, we have to go to second order in λ to have a one loop integral that contributes to the three-point function, so it will be suppressed with a factor of λ compared to the two-point seagull contribution. Going to a $\lambda\phi^6$ type interaction, the suppression is H^2/M_p^2 compared to the two-point seagull contribution, from dimensional considerations.

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A Inflaton perturbations in the ADM formalism

It is convenient to use the ADM formalism [34] to derive the action for the inflaton perturbations. Let us consider the scalar action of the inflaton field

$$S = \frac{1}{2} \int \sqrt{g} [R - (\partial\phi)^2 - 2V(\phi)] , \quad (65)$$

in the ADM metric, given by

$$ds^2 = -\mathcal{N}^2 dt^2 + h_{ij}(dx^i + \mathcal{N}^i dt)(dx^j + \mathcal{N}^j dt) . \quad (66)$$

In this metric, the action becomes [34]

$$S = \frac{1}{2} \int \sqrt{h} \left[\mathcal{N} R^{(3)} - 2\mathcal{N}V + \mathcal{N}^{-1} (E_{ij}E^{ij} - E^2) + \mathcal{N}^{-1} \left(\dot{\phi} - \mathcal{N}^i \partial_i \phi \right)^2 - \mathcal{N} h^{ij} \partial_i \phi \partial_j \phi \right] , \quad (67)$$

where

$$E_{ij} = \frac{1}{2} \left(\dot{h}_{ij} - \nabla_i \mathcal{N}_j - \nabla_j \mathcal{N}_i \right) . \quad (68)$$

As mentioned in the introduction, we find it convenient to discuss the effective action of the inflaton perturbations in the uniform curvature gauge, where, when ignoring vector and tensor modes, we have

$$\phi = \phi_c + \delta\phi , \quad h_{ij} = a^2 \delta_{ij} , \quad \mathcal{N} = 1 + \alpha , \quad \mathcal{N}^i = \partial_i \chi . \quad (69)$$

The strength of the ADM formalism, is that the constraint equations are easily obtained by varying the action in N and N_i , which acts as Lagrange multipliers. In this way the constraint equations in the uniform curvature gauge becomes

$$-a^2 \delta^{ij} \partial_i \phi \partial_j \phi - 2V - \mathcal{N}^{-2} \left(E_{ij}E^{ij} - E^2 + \left(\dot{\phi} - \mathcal{N}^i \partial_i \phi \right)^2 \right) = 0 , \quad (70)$$

and

$$\nabla_j [\mathcal{N}^{-1} (E_j^i - \delta_j^i E)] = \mathcal{N}^{-1} \left(\dot{\phi} - \mathcal{N}^j \partial_j \phi \right) \partial_i \phi . \quad (71)$$

If we perturb the action by taking

$$\phi = \phi_c + \delta\phi , \quad \alpha = \alpha_1 + \alpha_2 + \dots , \quad \chi = \chi_1 + \chi_2 + \dots , \quad (72)$$

and solve the constraints equations order by order, one finds to first order [35]

$$\alpha_1 = \frac{1}{2} \frac{\dot{\phi}_c}{H} \delta\phi, \quad \partial^2 \chi_1 = -\frac{1}{2} \frac{\dot{\phi}_c}{H} \delta\dot{\phi} - \frac{1}{2} \dot{\phi}_c \frac{\dot{H}}{H} \delta\phi + \frac{1}{2} \frac{\ddot{\phi}}{H} \delta\phi. \quad (73)$$

Generally, in order to obtain the action to order n , we need only to derive the constraint equations to order $n-1$, since the n 'th order terms multiplies the constraint equation to zero'th order. In fact, it also turns out that to obtain the action to third order in perturbations, one only needs the first order terms in eq.(73), since α_2 , χ_2 cancels out to leading order in the slow-roll expansion. One obtains

$$\begin{aligned} S_3 = \int a^3 \left[-\frac{1}{4} \frac{\dot{\phi}_c}{H} \delta\dot{\phi}^2 \delta\phi - \frac{a^2}{4} \frac{\dot{\phi}_c}{H} \delta\phi (\partial\delta\phi)^2 - \delta\dot{\phi} \partial_i \chi_1 \partial_i \delta\phi \right. \\ \left. + \frac{3}{8} \frac{\dot{\phi}_c^3}{H} \delta\phi^3 - \frac{1}{4} \frac{\dot{\phi}_c}{H} V'' \delta\phi^3 - \frac{1}{6} V''' \delta\phi^3 + \frac{1}{4} \frac{\dot{\phi}_c^3}{H^2} \delta\phi^2 \delta\dot{\phi} + \frac{1}{4} \frac{\dot{\phi}_c^2}{H} \delta\phi^2 \partial^2 \chi_1 \right. \\ \left. + \frac{1}{4} \frac{\dot{\phi}_c}{H} (-\delta\phi \partial_i \partial_j \chi_1 \partial_i \partial_j \chi_1 + \delta\phi \partial^2 \chi_1 \partial^2 \chi_1) \right], \quad (74) \end{aligned}$$

as first derived by Maldacena [35], and subsequently generalized in Ref. [36–38]. By going one order further, we can in a similar fashion obtain the action to fourth order in perturbations

$$\begin{aligned} S_4 = \int a^3 \left[\frac{1}{16} \frac{\dot{\phi}_c^2}{H^2} \delta\dot{\phi}^2 \delta\phi^2 - \frac{a^2}{16} \frac{\dot{\phi}_c^2}{H^2} \delta\phi^2 (\partial\delta\phi)^2 + \frac{1}{2} \frac{\dot{\phi}_c}{H} \delta\phi \delta\dot{\phi} \partial_i \chi_1 \partial_i \delta\phi + \frac{1}{24} \frac{\dot{\phi}_c^3}{H^2} \delta\phi^3 \partial^2 \chi_1 \right. \\ \left. - \frac{15}{64} \frac{\dot{\phi}_c^4}{H^2} \delta\phi^4 - \frac{1}{16} \frac{\dot{\phi}_c^2}{H^2} V'' \delta\phi^4 - \frac{1}{12} \frac{\dot{\phi}_c}{H} V''' \delta\phi^4 - \frac{1}{24} V'''' \delta\phi^4 + \frac{1}{2} (\partial_i \chi_1)^2 (\partial_j \delta\phi)^2 \right. \\ \left. - \delta\dot{\phi} \partial_i \chi_2 \partial_i \delta\phi - \frac{1}{16} \frac{\dot{\phi}_c^2}{H^2} ((\partial^2 \chi_1)^2 - (\partial_i \partial_j \chi_1)^2) \delta\phi^2 - \frac{1}{2} ((\partial^2 \chi_2)^2 - (\partial_i \partial_j \chi_2)^2) \right. \\ \left. + \frac{\dot{\phi}_c}{2H} (\delta\phi \partial^2 \chi_1 \partial^2 \chi_2 - \delta\phi \partial_i \partial_j \chi_1 \partial_i \partial_j \chi_2) - \partial^2 \chi_1 \partial^2 \chi_3 + \partial_i \partial_j \chi_1 \partial_i \partial_j \chi_3 \right. \\ \left. - \left(2H \partial^2 \chi_2 + \frac{1}{2} V'' \delta\phi^2 + \frac{1}{2} (\partial\delta\phi)^2 + \frac{1}{4} \frac{\dot{\phi}_c^2}{H^2} V \delta\phi^2 + V F(\delta\phi, \dot{\delta\phi}) \right) \right. \\ \left. - \frac{1}{2} (\partial^2 \chi_1)^2 + \frac{1}{2} (\partial_i \partial_j \chi_1)^2 + \frac{1}{2} \delta\dot{\phi}^2 - \dot{\phi}_c \partial_i \chi_1 \partial_i \delta\phi \right) F(\delta\phi, \dot{\delta\phi}) \Big], \quad (75) \end{aligned}$$

valid up to total derivative terms. We also note that to leading order in slow-roll α_3 has cancelled out of the action. Above, we also used the solution to the constraint equations to second order,

$$\alpha_2 = \frac{\dot{\phi}_c^2}{8H^2} + F(\delta\phi, \dot{\delta\phi}), \quad (76)$$

and

$$\partial^2 \chi_2 = \frac{3}{8} \frac{\dot{\phi}_c^2}{H} \delta\phi^2 + \frac{3}{4} \frac{\ddot{\phi}_c}{H \dot{\phi}_c} \delta\phi^2 - \frac{a^2}{4H} (\partial\delta\phi)^2 - \frac{1}{4H} \delta\dot{\phi}^2 + \frac{\dot{\phi}_c}{2H} \partial_i \chi_1 \partial_i \delta\phi$$

$$+\frac{1}{4H}((\partial^2\chi_1)^2-(\partial_i\partial_j\chi_1)^2)-\frac{V}{H}F(\delta\phi,\dot{\delta\phi}) , \quad (77)$$

where we have for convenience defined

$$F(\delta\phi,\dot{\delta\phi})=\frac{1}{2H}\partial^{-2}\left[\partial^2\alpha_1\partial^2\chi_1-\partial_i\partial_j\alpha_1\partial_i\partial_j\chi_1+\partial_i\dot{\delta\phi}\partial_i\delta\phi+\dot{\delta\phi}\partial^2\delta\phi\right] . \quad (78)$$

The extra terms in the S_4 action involving α_3 , which cancels to leading order in slow-roll, are $(2\alpha_1\alpha_2-\alpha_3)(4H\partial^2\chi_1+2\dot{\phi}_c\dot{\delta\phi})$, $-(V'+\frac{\dot{\phi}_c}{H}V)\delta\phi\alpha_3$. By using V to leading order in slow-roll we also neglected a term $(5/128)(\dot{\phi}_c^6/H^4)\delta\phi^4$ in the action S_4 , and in a similar fashion we have also neglected terms of higher order in slow-roll in the expression for χ_2 .

B IR scaling behavior

In this section of the appendix, we will evaluate the IR part of the correlator $(\delta\phi^2)_0$. We know that in the chosen gauge $|U_k|=(\dot{\phi}/H)\mathcal{R}_k$, where $\mathcal{R}_k=-\zeta_k$ is the gauge invariant curvature perturbation, constant on super-Hubble scales. Thus, we can solve the mode equations in pure de Sitter ($\epsilon=0$) and then evaluate \mathcal{R}_k , but using the value of $H=H_k$ and $\dot{\phi}=\dot{\phi}_k$ when the given mode crosses the horizon. Since we know that \mathcal{R}_k and U_k scales in the same way, we can first integrate over the power spectrum of \mathcal{R}_k and then relate that to the two-point function of $\delta\phi$ at the end. In pure de Sitter on super-Hubble scales $|U_k(\eta)|=H/2\pi$, so the curvature perturbation in this approach can be written as

$$\mathcal{R}_k\simeq\frac{1}{2\pi}\left(\frac{H_k^2}{\dot{\phi}_k}\right) . \quad (79)$$

If we specify to the specific $\lambda\phi^4$ model of chaotic inflation, this implies that we obtain the right k scaling from

$$U_k(\eta)=\left(\frac{H_k}{H(\eta)}\right)^{3/2}U_k^{ds}(\eta)=\frac{\sqrt{\pi}}{2}H\eta^{3/2}\left(\frac{H_k}{H(\eta)}\right)^{3/2}H_{3/2}^{(2)}(k\eta) , \quad (80)$$

where the mode solution in pure de Sitter was denoted by $U_k^{ds}(\eta)$.

If we let $N=\ln(a/a_i)$ denote the number of e-foldings of expansions since the beginning of inflation, it is convenient to recast the relevant integral into an integral over N . To do so, we use the relations $d\ln k=dN$, $\ln(aH/k)=N$ and

$$H_k^2=\frac{\lambda}{12M_p^2}(\phi_i^2-64M_p^2N)^2 . \quad (81)$$

Now we can easily evaluate the dominant part of the IR contribution to the relevant correlator

$$\begin{aligned} \langle |\delta\phi(t,\mathbf{x})|^2 \rangle_{IR} &\simeq \frac{H^2}{4\pi^2} \int_{a_i H_i}^{aH} \frac{dk}{k} \left(\frac{H_k}{H}\right)^3 \\ &= \frac{H^2}{4\pi^2} \left(\frac{\lambda}{12M_p^2 H^2}\right)^{3/2} \int_0^N dN (\phi_i^2 - 64M_p^2 N)^3 \end{aligned} \quad (82)$$

Using $N \simeq \phi_i^2/(64M_p^2)$, we obtain

$$\begin{aligned} \langle |\delta\phi(t, \mathbf{x})|^2 \rangle_{IR} &\simeq \frac{H^2}{4\pi^2} \left(\frac{\lambda}{12M_p^2 H^2} \right)^{3/2} \frac{1}{256M_p^2} \phi_i^8 \\ &= \frac{H^2}{16\pi^2} \left(\lambda \frac{1024}{3} \frac{M_p^2}{H^2} \right)^{3/2} N^4 . \end{aligned} \quad (83)$$

As mentioned also in section (2.1), our results are consistent with [15–17], when comparison is possible.

C Seagull integral

Consider the seagull integral in eq.(57). Instead of using the approximation in eq.(58), we want to use the exact expression with the correct scaling behavior

$$U_k(\eta) = \frac{\sqrt{\pi}}{2} H(-\eta)^{3/2} H_\nu^{(2)}(-k\eta) . \quad (84)$$

In the approximation where we can treat $H^2\Delta_N$ as constant, the relevant integral becomes

$$\begin{aligned} I_1(\eta_0, k) &= \int \frac{d\eta}{\eta^4 H^4} \text{Im} [U_k^2(\eta_0) U_k^{*2}(\eta)] \\ &= \frac{\pi^2}{16} \int d\eta \eta^2 \text{Im} [H_\nu^{(2)}(-k\eta_0) H_\nu^{(2)}(-k\eta_0) H_\nu^{(1)}(-k\eta) H_\nu^{(1)}(-k\eta)] . \end{aligned} \quad (85)$$

It is useful to make a transformation to a dimensionless parameter $x = -k\eta$, such that the integral becomes

$$I_1(x_0, k) = -\frac{\pi^2}{16} \int dx x^2 \text{Im} [H_\nu^{(2)}(x_0) H_\nu^{(2)}(x_0) H_\nu^{(1)}(x) H_\nu^{(1)}(x)] . \quad (86)$$

We know from our exact calculation in the case $\nu = 3/2$, that in the limit $x_0 \rightarrow 0$, the dominant contribution to the integral are a constant contribution from $x \simeq 1$ and a logarithmic x_0 dependent contribution from $x \ll 1$. That means, that we can obtain a good approximation to the integral by expanding the Hankel function in the small argument limit, reproducing the x_0 dependent contribution to the integral, and add the constant contribution computed in the $\nu = 3/2$ case. In this way we obtain

$$\begin{aligned} I_1(x_0, k) &= \frac{1}{k^3} \frac{\pi^2}{8} \frac{2^{2\nu}}{\sin^3(\nu\pi) \Gamma^3(1-\nu) \Gamma(\nu+1)} \left(\frac{1}{3} - \frac{1}{3-2\nu} \right) x_0^{3-2\nu} + \frac{20}{12} , \quad \text{for } \nu \neq 3/2 , \\ I_1(x_0, k) &= \frac{1}{k^3} \frac{\pi^2}{8} \frac{2^{2\nu}}{\Gamma^3(1-\nu) \Gamma(\nu+1)} \left(\frac{1}{3} - \ln(x_0) \right) + \frac{20}{12} , \quad \text{for } \nu = 3/2 . \end{aligned} \quad (87)$$

In fig.(4), we have compared the exact solution for the integral in the case $\nu = 3/2$ with the approximate solution given in eq.(87). We find that the approximation is reasonably good on very super-Hubble scales $x_0 \ll 10^{-3}$.

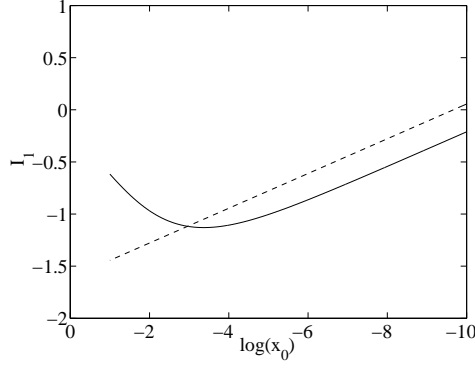


Figure 3: Different approximations to the integral $I_1(x_0)$ in $T^{(1)}(k)$. The solid line is the solution calculated in section 3.2 and the dashed line is the solution calculated above in eq.(87).

D One loop diagrams

The Bubble diagram, $T_1^{(2)}$ shown in fig.(4), is given by

$$T_1^{(2)}(k, \eta_0) = 36\lambda^2 \int_{-\infty}^{\eta_0} d\eta \int \frac{d^3 y}{\eta^4 H^4} \int_{-\infty}^{\eta_0} d\eta' \int \frac{d^3 y'}{\eta'^4 H'^4} \phi_c(\eta) \phi_c(\eta') [(G^>(x_1, y) - G^<(x_1, y)) \times (G^>(x_2, y')G^>(y, y')G^>(y, y') - G^<(x_2, y')G^<(y, y')G^<(y, y'))] . \quad (88)$$

The diagram $T_2^{(2)}$, shown in fig.(5), is given by

$$T_2^{(2)}(k, \eta_0) = 24(48\lambda)^2 \int_{-\infty}^{\eta_0} d\eta \int \frac{d^3 y}{\eta^4 H^4} \int_{-\infty}^{\eta_0} d\eta' \int \frac{d^3 y'}{\eta'^4 H'^4} \phi_c(\eta) \phi_c(\eta') \times [(G^>(x_1, y)G^>(x_2, y') - G^<(x_1, y)G^<(x_2, y')) \times (G^>(y, y')G^>(y', y') - G^<(y, y')G^<(y', y'))] . \quad (89)$$

However, since the tadpole subgraph is canceled by the tadpole renormalization condition, this diagram will also cancel out.

Finally there is three two loop diagrams contributing to second order in λ . They are labeled $\tilde{T}_1^{(2)}$, $\tilde{T}_2^{(2)}$, $\tilde{T}_3^{(2)}$ in fig.(7), and are given below

$$\tilde{T}_1^{(2)}(k, \eta_0) = 3(48\lambda)^2 \int_{-\infty}^{\eta_0} d\eta \int \frac{d^3 y}{\eta^4 H^4} \int_{-\infty}^{\eta_0} d\eta' \int \frac{d^3 y'}{\eta'^4 H'^4} \times [(G^>(x_1, y)G^>(x_2, y') - G^<(x_1, y)G^<(x_2, y')) \times (G^>(y, y')G^>(y, y')G^>(y', y') - G^<(y, y')G^<(y, y')G^<(y', y'))] . \quad (90)$$

$$\tilde{T}_2^{(2)}(k, \eta_0) = 4(48\lambda)^2 \int_{-\infty}^{\eta_0} d\eta \int \frac{d^3 y}{\eta^4 H^4} \int_{-\infty}^{\eta_0} d\eta' \int \frac{d^3 y'}{\eta'^4 H'^4}$$

$$\begin{aligned}
& \times [(G^>(x_1, y)G^>(y, y) - G^<(x_1, y)G^<(y, y)) \\
& \times (G^>(x_2, y')G^>(y, y')G^>(y', y') - G^<(x_2, y')G^<(y, y')G^<(y', y'))] \quad . \quad (91)
\end{aligned}$$

$$\begin{aligned}
\tilde{T}_3^{(2)}(k, \eta_0) &= 4(48\lambda)^2 \int_{-\infty}^{\eta_0} d\eta \int \frac{d^3 y}{\eta^4 H^4} \int_{-\infty}^{\eta_0} d\eta' \int \frac{d^3 y'}{\eta'^4 H'^4} \\
&\times [(G^>(x_1, y)G^>(y, y') - G^<(x_1, y)G^<(y, y')) \\
&\times (G^>(x_2, y')G^>(y, y')G^>(y, y') - G^<(x_2, y')G^<(y, y')G^<(y, y'))] \quad . \quad (92)
\end{aligned}$$

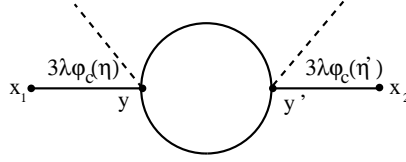


Figure 4: The bubble diagram $T_1^{(2)}$.

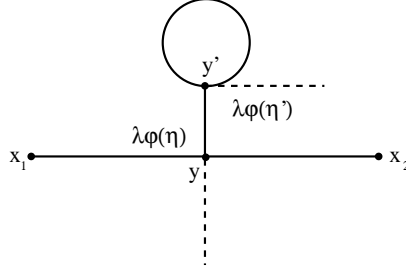


Figure 5: The tadpole sub-diagram in the $T^{(2)}$ contribution above, is canceled to higher order by the tadpole renormalization condition.

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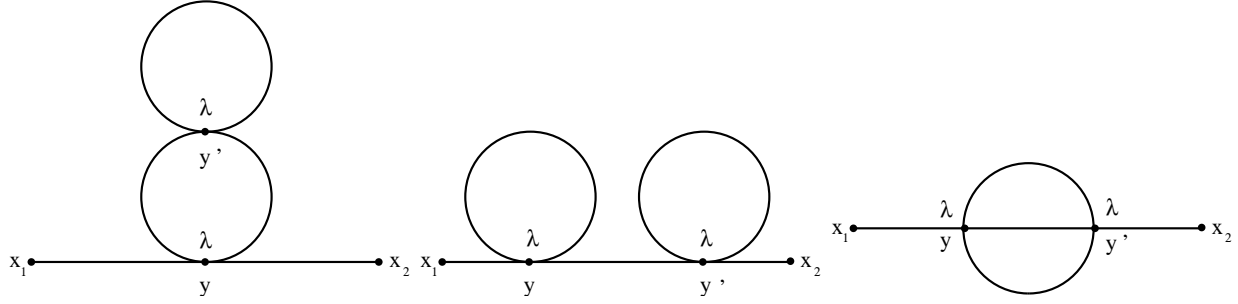


Figure 6: The two loop contributions, contributing to second order in λ .

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